Hysteresis & Return Point Memory in the Random Bond Ising Model in 2D with Moore Neighbourhood

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Abstract

The results of numerical simulations showing hysteresis in the Random Bond Ising model at T = 0 and in two dimensions resulting from interactions with its Moore neighbourhood are presented for the first time. The phenomena of Return Point Memory is also examined. The results presented are the initial simulations and stochastic effects are not taken into account.

1 Introduction

The Lenz-Ising Model (IM) [1] is the simplest model for study of characteristics arising only from spin. Extension of the IM using random fields [2,3] and random bonds [4, 5] has been used to show hysteresis in the model. However, in these models the interaction were limited to the nearest Von-Neumann neighbours only. Here the results of numerical simulations show that hysteresis in the Random Bond Ising Model (RBIM) is also present in the case of interactions due to the Moore neighbourhood too. The phenomena - return hysteresis loop [6] which is also known as return point memory (RPM) is also examined in these simulations.

2 RBIM with Moore Neighbourhood (MN)

The RBIM is an array of lattice sites with each site exhibiting either spin up or down $(\sigma_{i,j} = \pm 1)$ In between each site to its neighbouring site the interaction is communicated through a Gaussian distribution of random bonds with strength limited up to ±1 only. As is usually done, the edge lattice points are reconnected to the "opposite" sides of the same lattice to avoid edge effects of a finite lattice size.

With the Moore neighbourhood in 2 dimensions each lattice site has 8 nearest neighbours. For the site labelled as (i,j), it will be connected to the sites (i-1,j-1),(i-1,j),(i-1,j+1),(i,j-1), (i,j+1),(i+1,j-1),(i+1,j),(i+1,j+1) as shown in the figure (1).

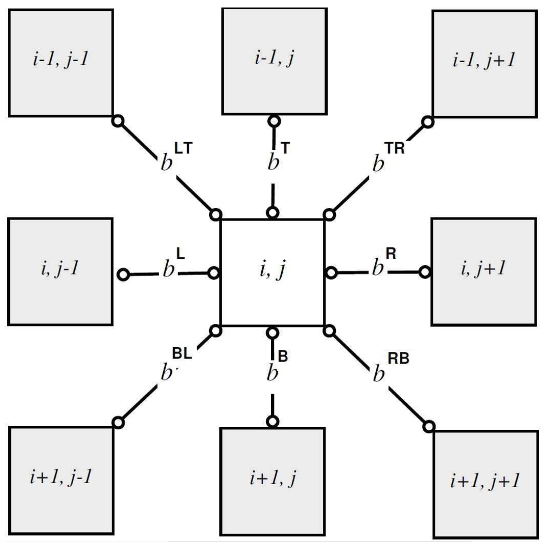

Figure 1: Moore Neighbourhood of (i, j) in 2 dimensions with the random bonds.

For a lattice sized i,j=1,2…n, the edge effects are taken care by

$$i+1 \to 1 \quad \forall \quad i = n, \qquad j+1 \to 1 \quad \forall \quad j = n$$

and,

$$i-1 \to n \quad \forall \quad i = 1, \qquad j-1 \to n \quad \forall \quad j = 1,$$

Figure (2) is a pictorial representation for lattice site (1,*n*) with its Moore Neighbours.

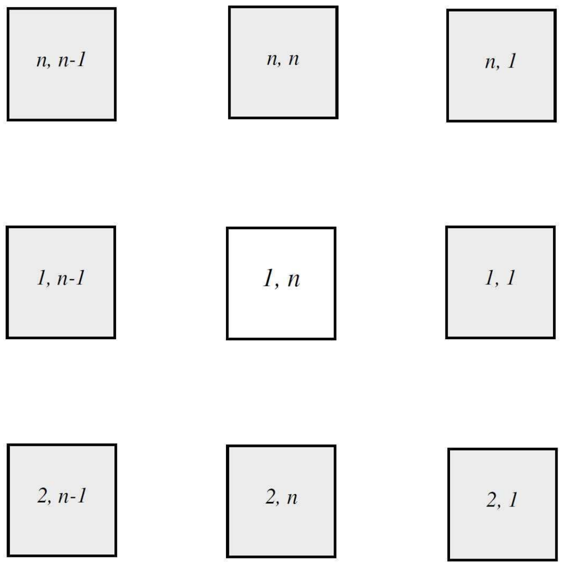

Figure 1: Moore Neighbourhood of (1,*n*).

Each lattice site (*i,j*) would be connected to each of its eight nearest neighbours via eight bonds of random strength between [-1,1] and their strength distribution is Gaussian. The bonds to its top, top-right, right, right-bottom, bottom, bottom-left, left, left-top are labelled as

$$b^{T}_{i-1,j}, b^{TR}_{i-1,j+1}, b^{R}_{i,j+1}, b^{RB}_{i+1,j+1}, b^{B}_{i+1,j}, b^{BL}_{i+1,j-1}, b^{L}_{i,j-1}, b^{LT}_{i-1,j-1} \quad (1)$$

respectively. These are illustrated in figure (1).

3 Dynamics of RBIM

In this model of the RBIM with MN, we assume that the spin-state of the (*i,j*)-th lattice site is decided by the sum of the interactions of its 1st degree Moore Neighbourhood. This is computed as,

$$\sigma_{i,j} = \begin{cases} +1 & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} > 0 \\ \sigma_{i,j} & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} = 0 \\ -1 & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} < 0. \end{cases} \quad (2)$$

In the above, the summation is over the Moore neighbourhood of the lattice site. The $<k>$ in the superscript of $b^{<k>}_{<i,j>}$ is to used to indicate the operation direction of the bond and would contain one of the identifiers T, TR, R, RB, B, BL, L or LT as the case may be.

In the presence of an uniform external field *H*, acting over the lattice, the spin-state of (*i,j*)-th site would now be dictated by,

$$\sigma_{i,j} = \begin{cases} +1 & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} + H > 0 \\ \sigma_{i,j} & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} + H = 0 \\ -1 & \text{if} \quad \sum_{<i,j>} b^{<k>}_{<i,j>} \sigma_{<i,j>} + H < 0. \end{cases} \quad (3)$$

The simulations are carried out in the following manner -

1. The individual spins of each lattice site is generated. They take random values of either -1 or +1. Consequently the magnetization, which is the sum total of their spins over the entire lattice is $M \approx 0$.
2. Next the random bonds $b^{<k>}_{<i,j>}$, as given by (1) which operate in between the nearest neighbours are enerated. Their values are restricted to [-1,+1] and have a Gaussian distribution to mimic natural phenomena.
3. The OA segment of the *M-H* curve in figure (3) is obtained by gradually increasing the external field *H* from 0 to a value of 6. For each value of *H*, each site of the lattice takes its spin depending on the dynamics (3). The process is repeated until the spin states in each of the lattice sites remain unchanged and equilibrium is achieved for this particular value of *H*. The value of *M* is calculated.
4. The process is repeated with the new incremented value of *H* and continued until *M* attains its maximum value of unity.
5. In a similar manner, the curves ABCD and DEFA are obtained by decrementing and incrementing *H* from points A and from D respectively.

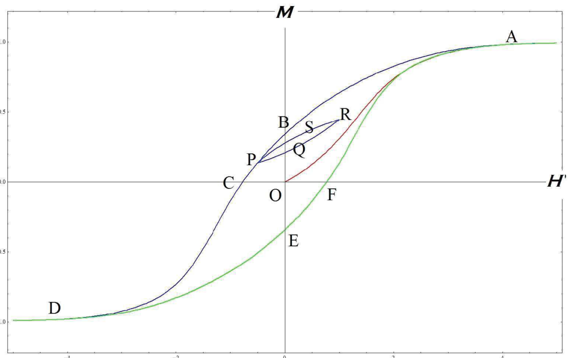

Figure 1: Hysteresis with RPM for n^2 = 500×500.

Return Point Memory (RPM)

As shown in (3) we can pause the decrement of *H* in the curve ABCD at any point P and start to increment *H* up to any value leading to point R via Q and again decrement *H* back to its original value at P. The curve RSP although different from PQR moves back to its initial point P. Further decrement of *H* makes the curve continue to D in the same manner as it would have done without the deviation at point P. The fact that *M* gets the same value at P for both PQR and RSP is called as the return point memory characteristic to the system.

4 Simulation Results

The simulation results for the curves OA, ABCD, DEFA for lattice size of 500×500 is reproduced in figure (3) and they clearly show the presence of hysteresis in the RBIM with MN. If the data set (*H*,*M*) from the numerical simulation for the curve DEFA set to the transformation , we get the curve ABCD. Thus the hysteresis curves are symmetric under such parity inversion.

The results for the RPM study are taken by considering P to be at *H*=-0.6 and R at *H*=1. The table (1) shows the difference of magnetization at P when arriving at it from A via B, and arriving at P via the loop PQRS for lattice sizes *n*=100,200,500. |Ä*M*| is the absolute difference of the magnetization. Its non-zero value signifies that the lattice is not identical at P before and after the loop PQRSP. By taking the ratio of |Ä*M*| with respect to the number of lattice sites n^2, we see that this relative difference becomes smaller as the lattice size increases. We can conclude that the RPM property is present in RBIM with the Moore Neighbourhood too and would become more exact for larger lattice sizes.

n	$\|\Delta M\|$	$\frac{\|\Delta M\|}{n^2}$
50	0.48	1.92×10^{-4}
100	0.52	5.2×10^{-5}
200	1.27	3.18×10^{-5}
500	0.46	1.83×10^{-5}

Table 1: Difference in the magnetization during the starting and ending of the RPM loop PQRSP at point P (at *H* = -0.6).

5 Conclusion

The simulation results presented here would help in giving guidelines to any theoretical explanations of such phenomena. Hysteresis is one of the simplest phenomena where the effects a changing environment, the external field, in a regular manner brings about a change to the system. And here it is demonstrated with one of the simplest of spin-systems, the RBIM with MN in two dimensions. We can conclude two features of the RBIM with MN; firstly the symmetry of the *M-H* curves during the complete reversal of magnetization and the presence of RPM. It is expected that any theory explaining hysteresis in this model should take into account these properties.

6 References

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